$$X_t \rightarrow \frac{X_t - \bar{X}}{s_X} = Y_t$$

Let s_X: sd of X

Tech. Ex 2 What is linear transformation behind standardization?

$$Y_t = a X_t + b$$

a, b have to be constants, do not depend on t (obs)

$$\frac{X_t - \bar{X}}{s_X} = \underbrace{\frac{1}{s_X}}_{a} X_t \underbrace{- \frac{\bar{X}}{s_X}}_{b}$$

$$\sum (X_t - \bar{X})(Y_t - \bar{Y}) = \sum (X_t - \bar{X}) Y_t$$

It does not say that

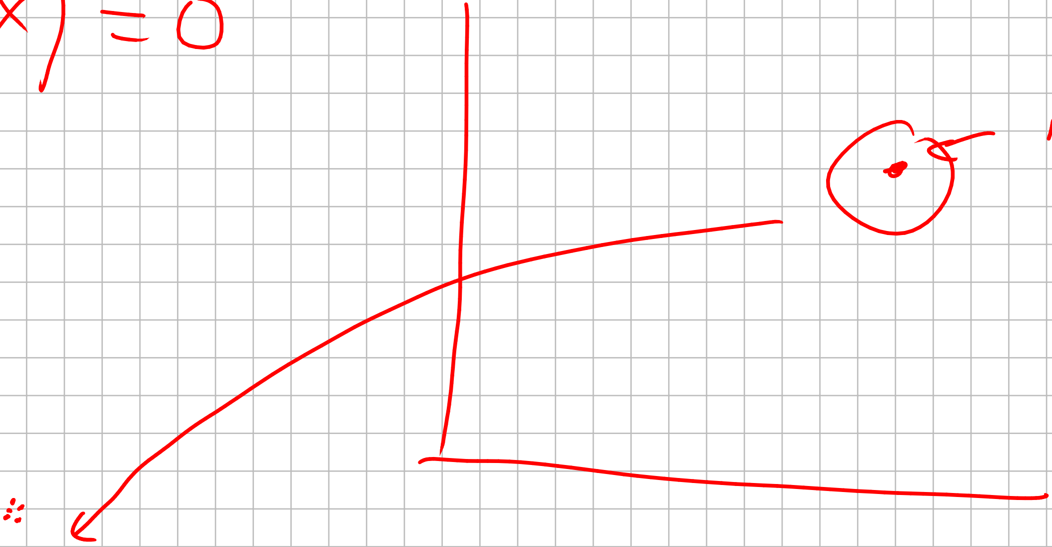
$$\sum (X_t - \bar{X})(Y_t - \bar{Y}) \doteq \sum X_t Y_t$$

t	X_t	Y_t	$X_t - \bar{X}$	$Y_t - \bar{Y}$	$(X_t-\bar{X})(Y_t-\bar{Y})$
1					
2					
⋮					
n					
means	$\bar{X}$	$\bar{Y}$	0	0	

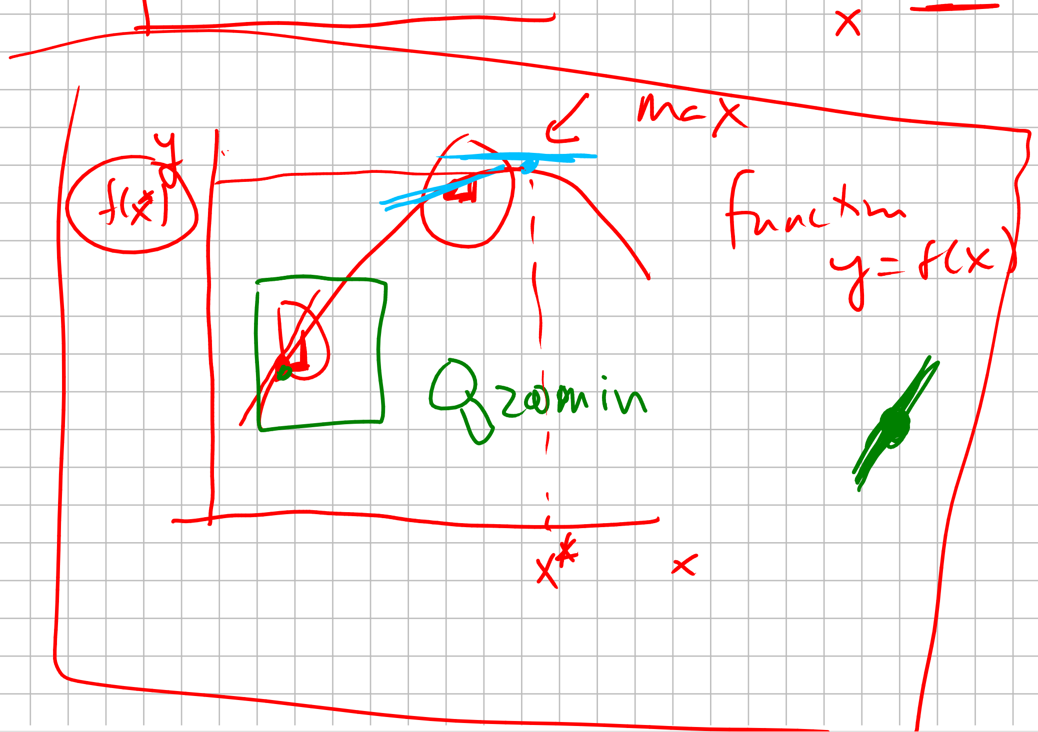
$$\sum_{t=1}^{n}(X_t - \bar{X}) = 0$$

$$\boxed{X_t} - \cancel{\bar{X}}$$

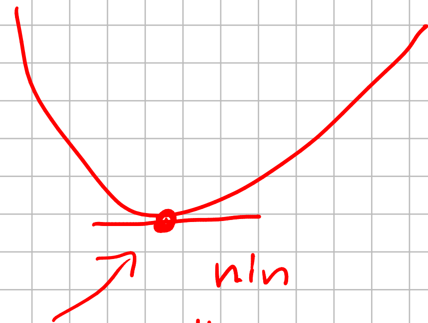
possible to have multiple observations share the same point



$\min\limits_{x} f(x)$ $\max\limits_{x} f(x)$



derivative = slopes of tangent lines



optimization for functions of a single variable

Here x is the choice variable

mean(subset(ratings$course_eval, ratings$female == 0)) ← averages for male

correlation coefficient of X & Y : $r_{X,Y}$

R-squared $R^2 = r_{X,Y}^2$ in the case where there is one regressor and an intercept

fitted values $\hat{Y}_t$

residuals $Y_t - \hat{Y}_t = e_t$ notation

variance of X $\frac{1}{n}\sum_{t=1}^{n}(X_t - \bar{X})^2$

variance of fitted values $\frac{1}{n}\sum_{t=1}^{n}(\hat{Y}_t - \bar{\hat{Y}})^2$

$$R^2 = \frac{\text{variance of fitted values}}{\text{variance of regressand}} \qquad \frac{\frac{1}{n}\sum_{t=1}^{n}(\hat{Y}_t - \bar{\hat{Y}})^2}{\frac{1}{n}\sum_{t=1}^{n}(Y_t - \bar{Y})^2}$$

$$= 1 - \frac{\text{variance of residuals}}{\text{variance of regressand}} = \ldots$$

$$= 1 - \frac{\sum_{t=1}^{n}(e_t - \bar{e})^2}{\sum_{t=1}^{n}(Y_t - \bar{Y})^2} = \ldots$$

Regressions with an intercept:

Can show that $\bar{e} = 0 \Rightarrow \sum_{t=1}^{n} e_t^2 = \sum_{t=1}^{n}(Y_t - \hat{Y}_t)^2$

Continue Tech Ex 3: Regression with only an intercept → what is R^2?

variance of fitted values $= \frac{1}{n}\sum_{t=1}^{n}\left(\boxed{\hat{Y}_t} - \bar{\hat{Y}}\right)^2$

↓

case in TE3

$= \frac{1}{n}\sum_{t=1}^{n}\left(\bar{Y} - \bar{Y}\right)^2 = 0$

Key

total variation / "explained" variation / "unexplained" variation

Variance of regressand = variance of fitted values + variance of residuals

$$\boxed{\sum (Y_t - \bar{Y})^2} = \boxed{\sum (\hat{Y}_t - \bar{\hat{Y}})^2} + \sum (e_t - \bar{e})^2$$

→ analysis of variance ANOVA

labor studies (earnings inequality)

finance studies (excess returns)

R^2 measure of fit

1, 2, 3 $\quad \bar{X} = 2$

lose a degree of freedom → 1, 2, $\underline{\quad}$ $\quad \bar{X} = 2$

more degrees of freedom → 1, ?, ? $\quad \bar{X} = 2$

X_t' ← transpose

$= (1, X_{1t}, X_{2t}, ..., X_{kt})$

t	Y_t	1	X_{1t}	X_{2t}	$\cdots$	X_{kt}	
1	Y_1	1	–	–	.	–	X_1'
2	Y_2	1					X_2'
3	–	1					
$\vdots$		$\vdots$					
n	Y_n	1					X_n'

$$\underset{(n\times 1)}{A} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \qquad \underset{(1\times n)}{A'} = (a_1 \ a_2 \ \ldots \ a_n) \qquad (A')' = A$$

$$\underset{((k+1)\times 1)}{\hat\beta} = (\hat\beta_0, \hat\beta_1, ..., \hat\beta_k)' = \begin{pmatrix} \hat\beta_0 \\ \hat\beta_1 \\ \hat\beta_2 \\ \vdots \\ \hat\beta_k \end{pmatrix}$$

$$\hat Y_t = X_t' \hat\beta = \underbrace{(\boxed{1}, X_{1t}, X_{2t}, ..., X_{kt})}_{1\times(k+1)} \underbrace{\begin{pmatrix} \boxed{\hat\beta_0} \\ \hat\beta_1 \\ \vdots \\ \hat\beta_k \end{pmatrix}}_{(k+1)\times 1}$$

$X_{0t} = 1$

$$= \hat\beta_0 \cdot 1 + \hat\beta_1 X_{1t} + \hat\beta_2 X_{2t} + \cdots + \hat\beta_k X_{kt}$$

$$= \hat\beta_0 + \sum_{j=1}^{k} \hat\beta_j X_{jt}$$

determinant of A

$$A = \begin{pmatrix} \boxed{4} & 1 \\ 3 & 2 \end{pmatrix} \qquad A^{-1} = ? \quad \frac{1}{4(2)-3(1)} \begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}$$

$$\underset{\text{matrix multiplication}}{AA^{-1}} = I = \begin{pmatrix} \boxed{1} & & 0 \\ & \boxed{1} & \\ 0 & & \ddots \ \boxed{1} \end{pmatrix}$$

square matrix
number of rows = number of columns.

A^{-1} may not exist when determinant of A is equal to zero.

$$\begin{array}{l} \text{1st row} \rightarrow \\ \text{2nd row} \rightarrow \end{array} \begin{pmatrix} \cancel{4} & \cancel{1} \\ 2 & 1/2 \end{pmatrix}$$

$$\min \sum_{t=1}^{n} (Y_t - \hat{Y}_t)^2$$

$$\downarrow$$

$$(\hat{\beta}_0 + \hat{\beta}_1 X_{1t} + \dots + \hat{\beta}_k X_{kt})$$