

TE 1

$$\textcircled{1} \quad \hat{Y}_A = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \hat{\beta}_3 x_1 x_2$$

$$\textcircled{2} \quad \hat{Y}_B = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 (x_2 + 1) + \hat{\beta}_3 x_1 (x_2 + 1)$$

$$\textcircled{3} \quad \hat{Y}_B - \hat{Y}_A = \hat{\beta}_2 + \hat{\beta}_3 x_1 \quad \text{when } x_1 \neq 0$$

$$\text{when } x_1 = 0, \quad \hat{Y}_B - \hat{Y}_A = \hat{\beta}_2.$$

$\textcircled{4}$  We cannot give an interpretation of  $\hat{\beta}_3$  alone as the comparison between  $\hat{Y}_A$  and  $\hat{Y}_B$  depends on both  $\hat{\beta}_2$  and  $\hat{\beta}_3$ , along with the level chosen for  $x_1$ . Even if  $x_1 = 0$ ,  $\hat{\beta}_3$  does not appear at all.

**NOTE**  $\hat{\beta}_2$  can be interpreted alone though provided  $x_1 = 0$  but this interpretation may be silly. You have seen examples of silly interpretations, which also motivates centering the regressors.

$\textcircled{5}$  When we consider students who have not attended any of the classes at all, the ones who have prior college GPA two points higher will have average standardized final exam scores lower by  $0.47(2) = 0.94$ . This roughly translates to having original final exam scores lower by a standard deviation.

**NOTE** Best interpretation, but still silly.

$\textcircled{6}$  Students who have prior college GPA higher by 1 point will have average standardized final exam scores higher by  $\hat{\beta}_2 + \hat{\beta}_3(80) \approx 0.62$ .

**NOTE** The answer here uses  $\textcircled{4}$ . The result here is less silly than  $\textcircled{5}$ .

TE 2

(1)

$\hat{\beta}_0$  is the average of the  $Y$ 's for those observations where  $X_{1t} = X_{2t} = 0$ .

$\hat{\beta}_1 = (\hat{\beta}_0 + \hat{\beta}_1) - (\hat{\beta}_0)$  is the difference in the average  $Y$  for the subgroup where  $X_{1t} = 1, X_{2t} = 0$  relative to the subgroup where  $X_{1t} = 0, X_{2t} = 0$ .

$\hat{\beta}_2 = (\hat{\beta}_0 + \hat{\beta}_2) - (\hat{\beta}_0)$  is the difference in the average  $Y$  for the subgroup where  $X_{1t} = 0, X_{2t} = 1$  relative to the subgroup where  $X_{1t} = 0, X_{2t} = 0$ .

$$\hat{\beta}_3 = \underbrace{(\hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3) - (\hat{\beta}_0 + \hat{\beta}_2)}_{\text{difference in the average } Y \text{ for the subgroup where } X_{1t} = X_{2t} = 1 \text{ relative to the subgroup where } X_{1t} = 0, X_{2t} = 1} - \underbrace{[(\hat{\beta}_0 + \hat{\beta}_1) - \hat{\beta}_0]}_{\text{difference in average } Y \text{ for the subgroup where } X_{1t} = 1, X_{2t} = 0 \text{ relative to subgroup where } X_{1t} = 0, X_{2t} = 0}$$

NOTE  $\hat{\beta}_3$ , as interpreted here, is connected to the so-called difference-in-difference (DiD) approach to causal inference (in its basic form). What we have here is only descriptive as assumptions are required in order to produce causal inferences.

② No, it may or may not make sense. If these majors are coded as mutually exclusive categories (meaning a student cannot have two majors at the same time), then it would not make sense. If you have double majors, then it may make sense.

NOTE In the Philippines, double majors do exist. For example, I am both an economics major and an accountancy major. In the US context, double majors may be rarer (they do have minors).